Checking Linearizability Using Hitting Families

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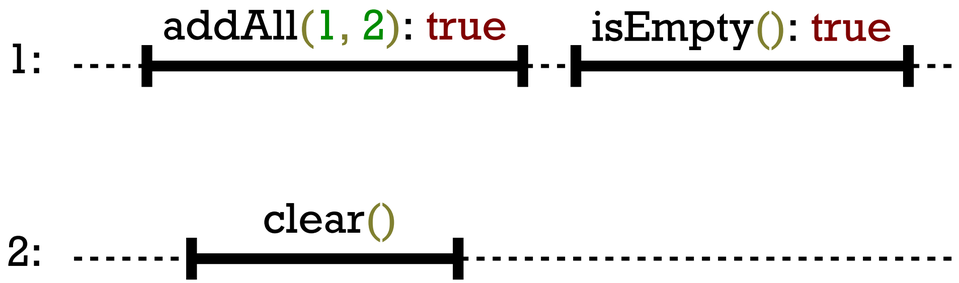
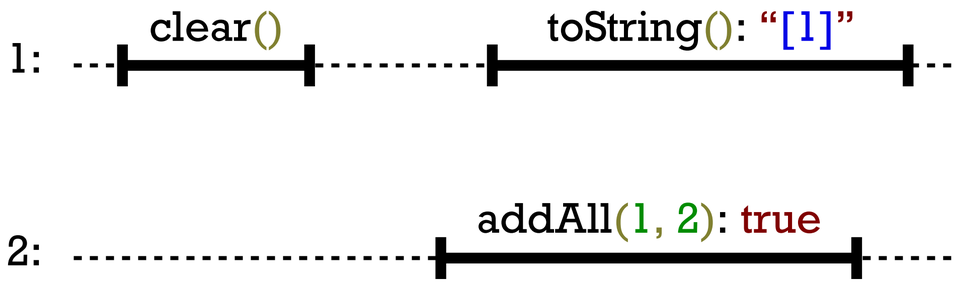
[2] University of Pennsylvania

Linearizability as a correctness condition

Two execution histories on a **concurrent list:**





Linearizable history: Concurrent operations can be totally ordered in a consistent way

Linearizable concurrent object: All of its execution histories are linearizable

Linearizability as a correctness condition

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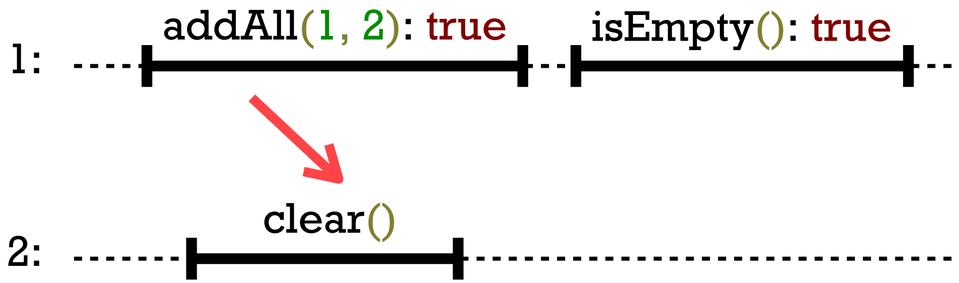
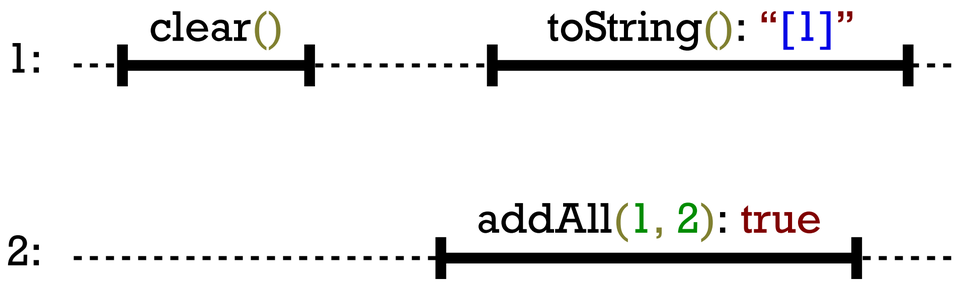


linearizable



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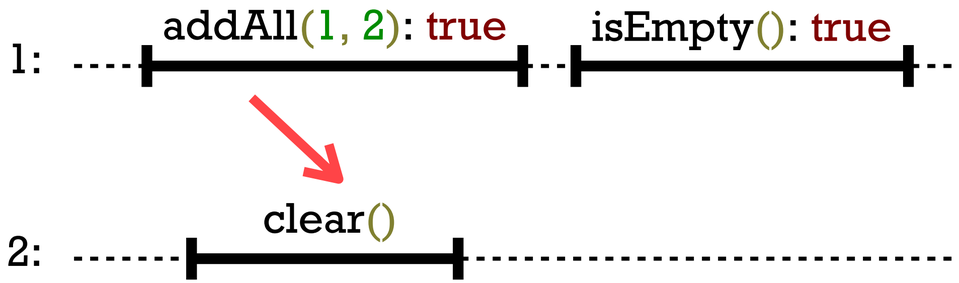

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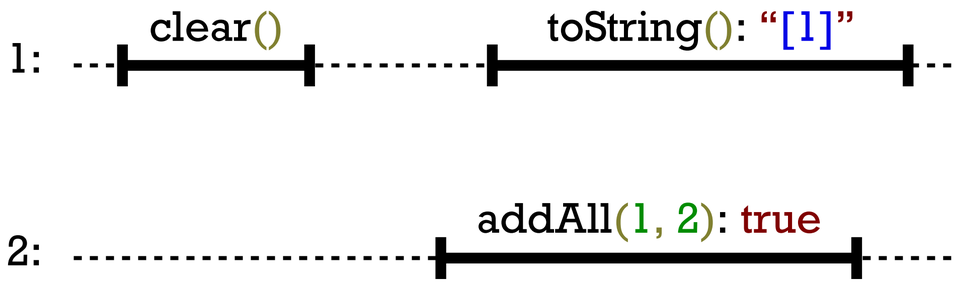

non-linearizable

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Checking linearizability

Linearizability of a concurrent object

- Pertains to **verification**
- **Undecidable**
- Approaches: Program logics, proof rules, semi-automated procedures

Linearizability of a single execution history

- Pertains to **testing**
- **NP-complete**
- Approaches: Exhaustive search for a linearizability witness with space-time trade-offs

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Our contribution

In a nutshell:

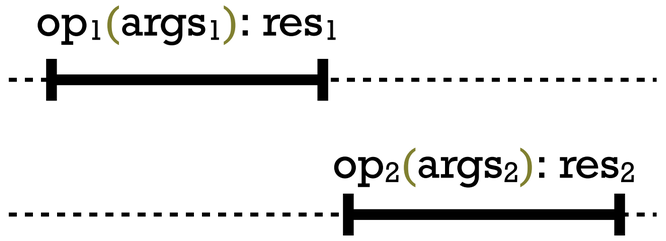
- Prioritize the search space to quickly find linearizability witnesses (if they exist)

In more detail:

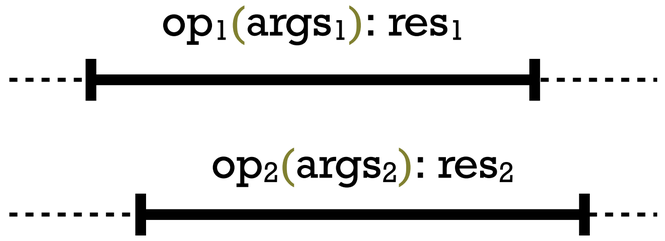
- Introduce **linearizability depth** of a history
- For a history of linearizability depth **d**, with **n** operations on **k** threads:
  Suffices to explore a **strong d-hitting family** of at most $\mathbf{O(kn^{d-1})}$ linearizations
- Experiments on **java.util.concurrent**:
  In most cases linearizability witnessed by strong **d**-hitting families with $\mathbf{d \leq 5}$!

Linearizability

Execution history induces a partial order of operations



op_1 happens before op_2



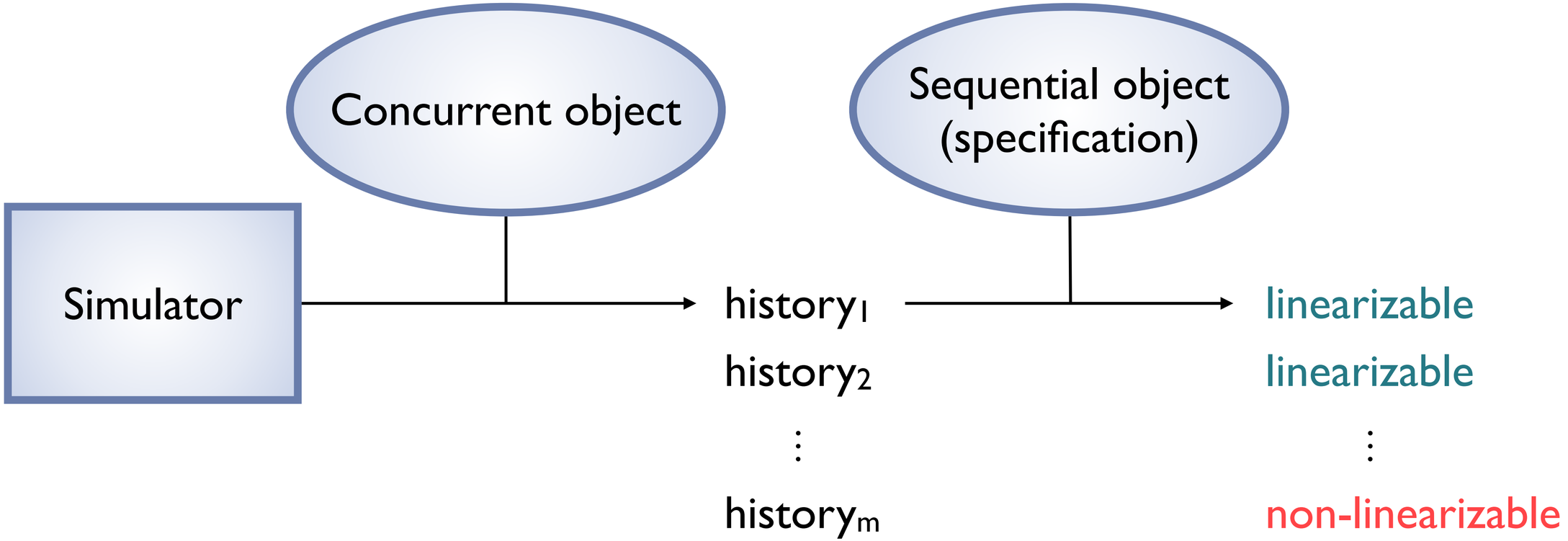
op_1 and op_2 are concurrent

Schedule: A total order of operations that extends the happens-before relation

Linearizability witness: A schedule in which the results of operations satisfy the sequential specification

Linearizable history: A history that has a linearizability witness

Testing concurrent objects [Wing and Gong '93]



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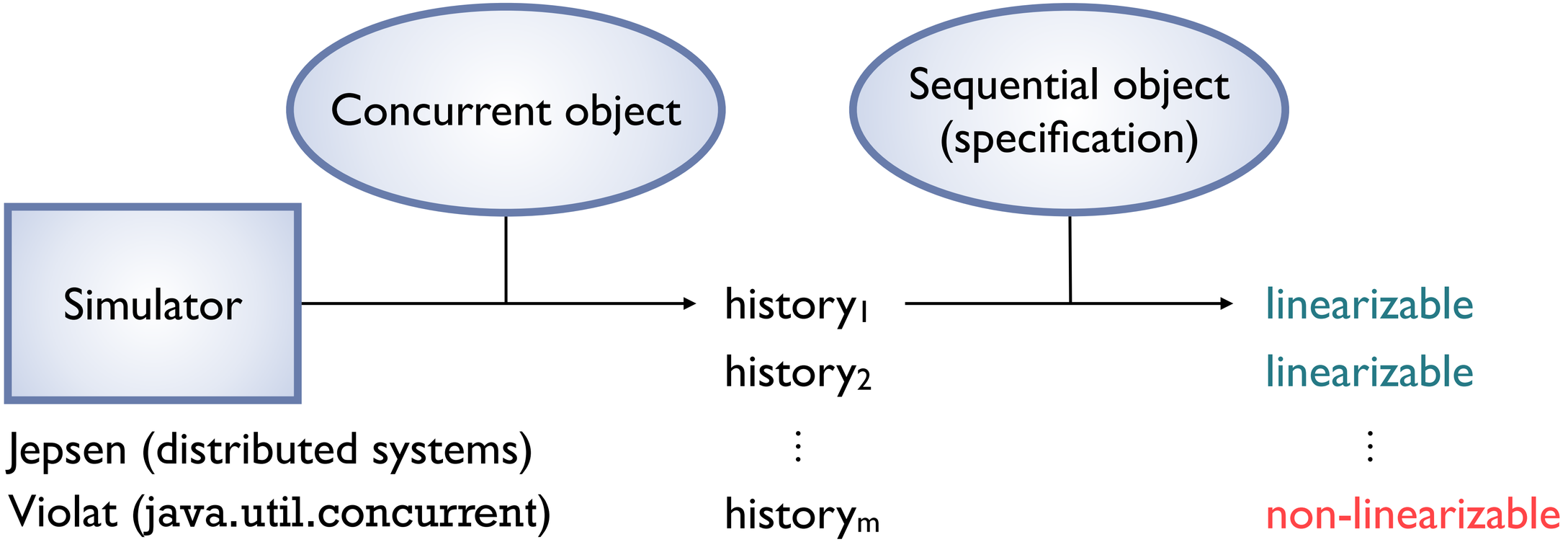

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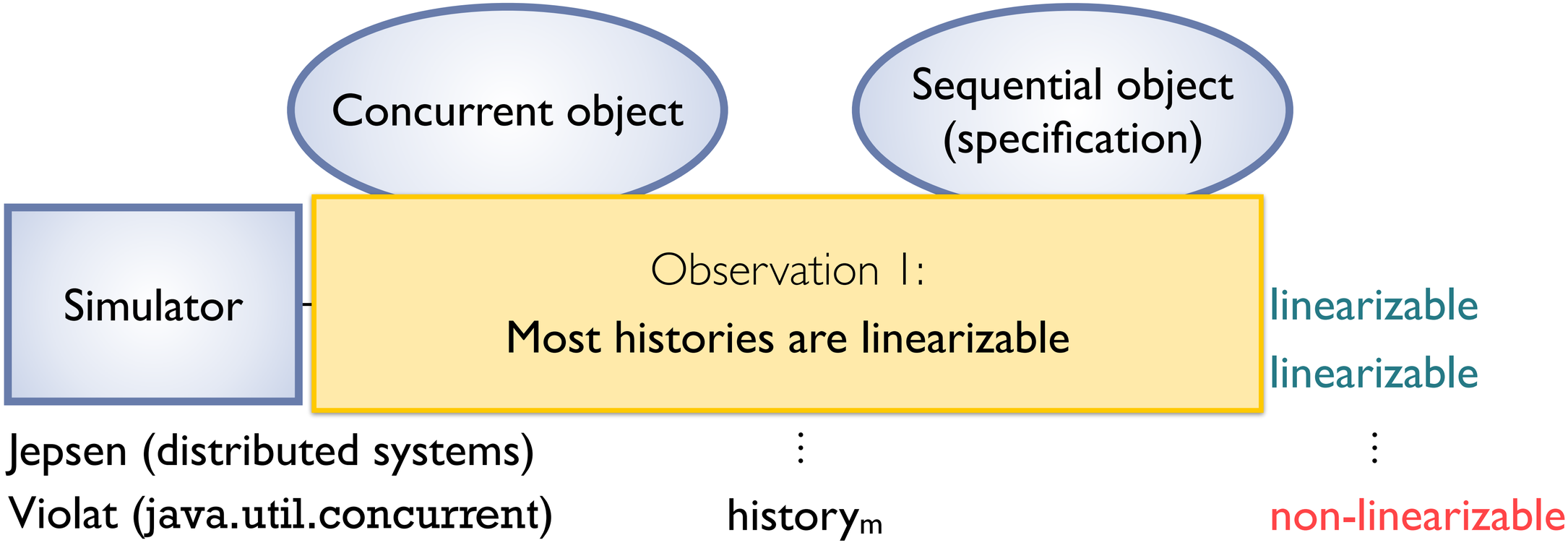
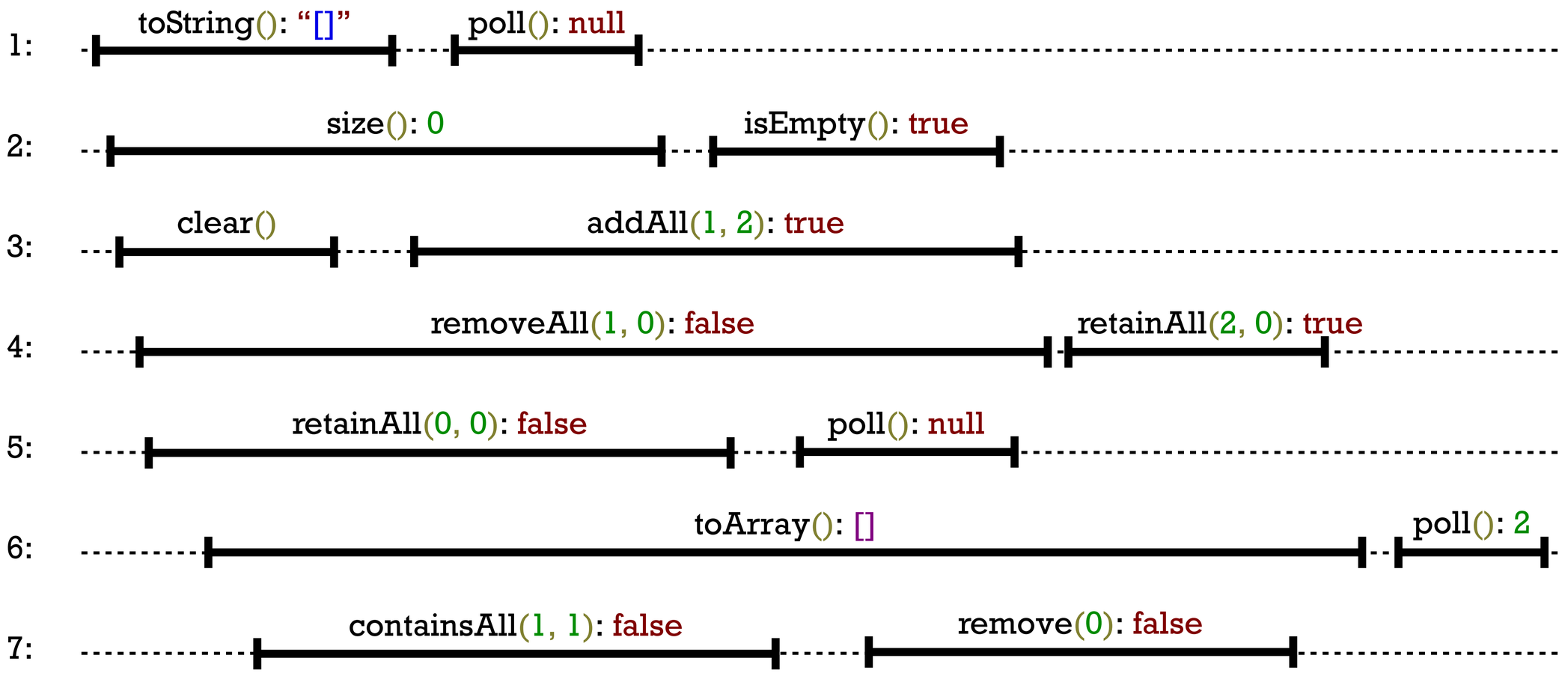


Execution history generated by Violat for ConcurrentLinkedQueue



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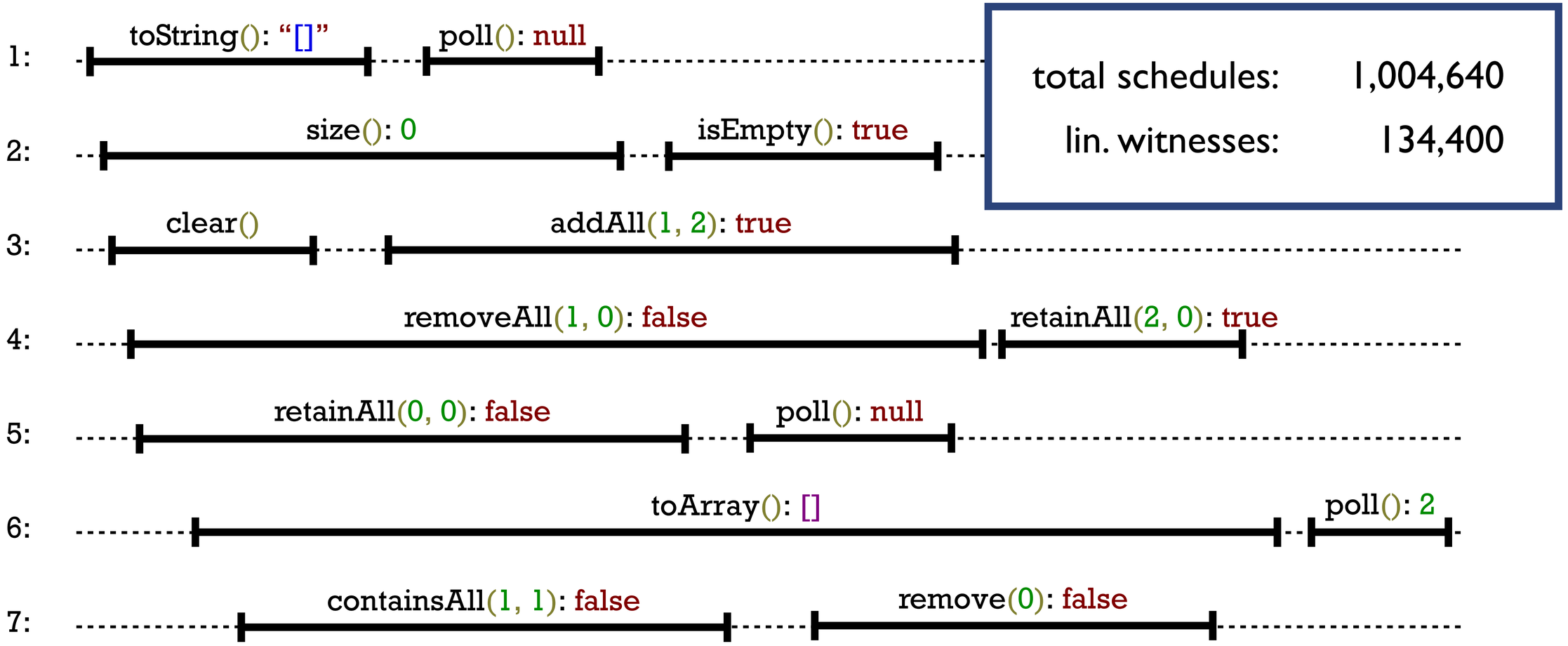

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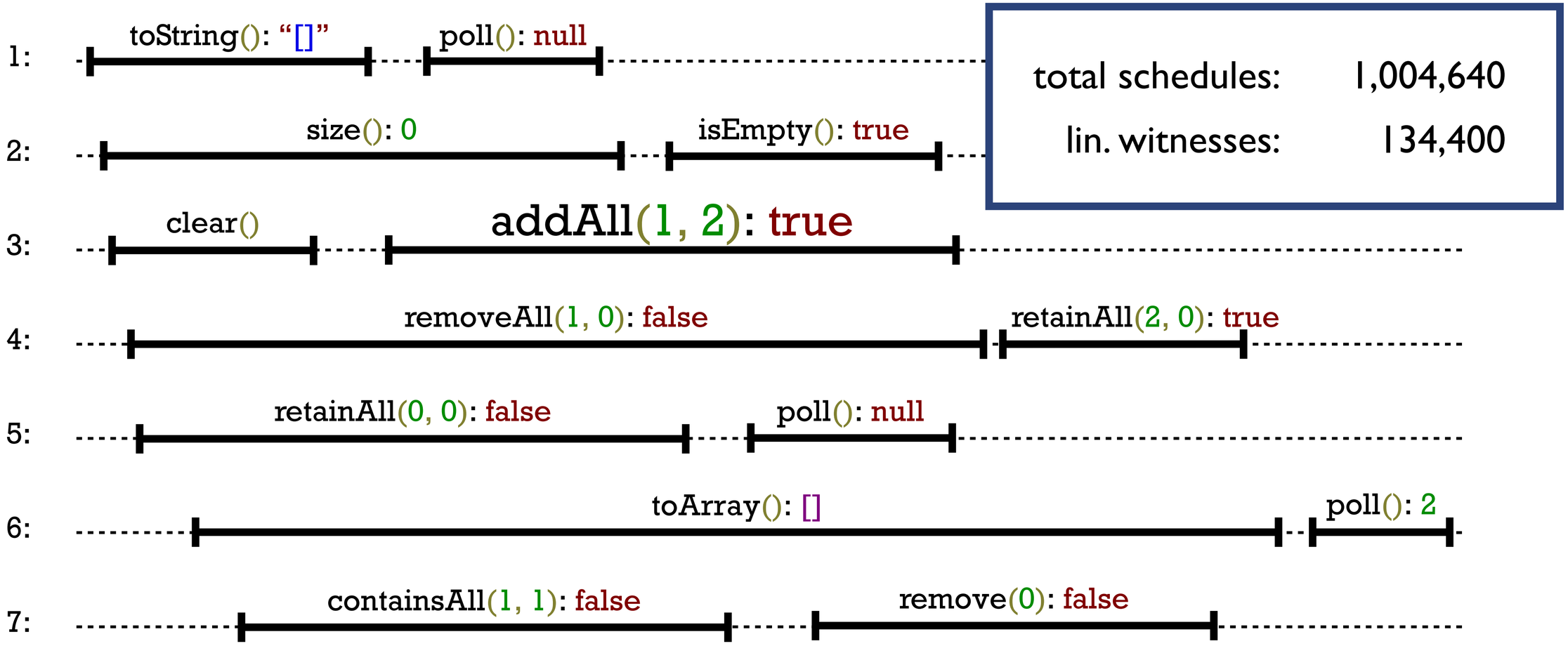

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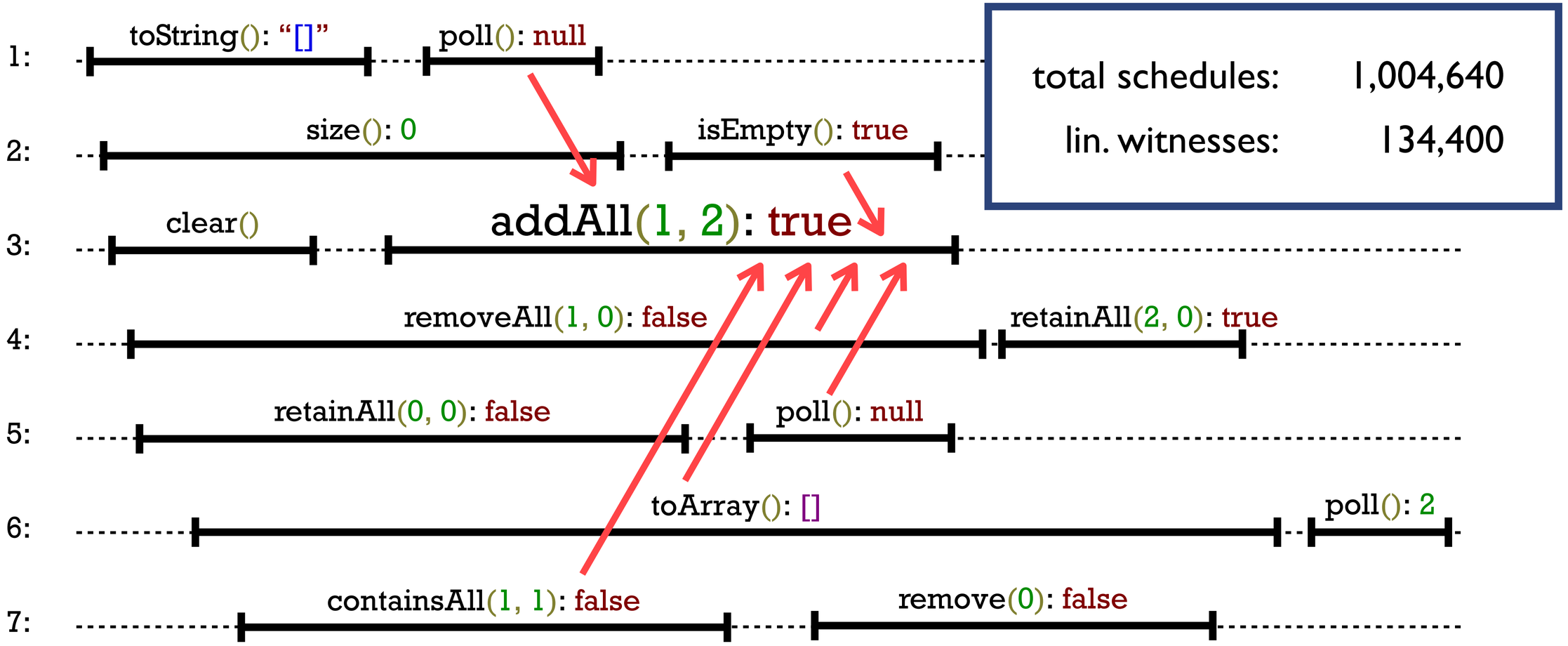


Linearizability depth

Strong hitting:

Given $\mathbf{d \geq 1}$, a schedule $\boldsymbol{\alpha}$ **strongly hits** a **d**-tuple of operations $\mathbf{(op_0, \ldots, op_{d-1})}$ if it

- maximally delays each $\mathbf{op_i}$
- while maintaining the relative order $\mathbf{op_0 <_\alpha \ldots <_\alpha op_{d-1}}$

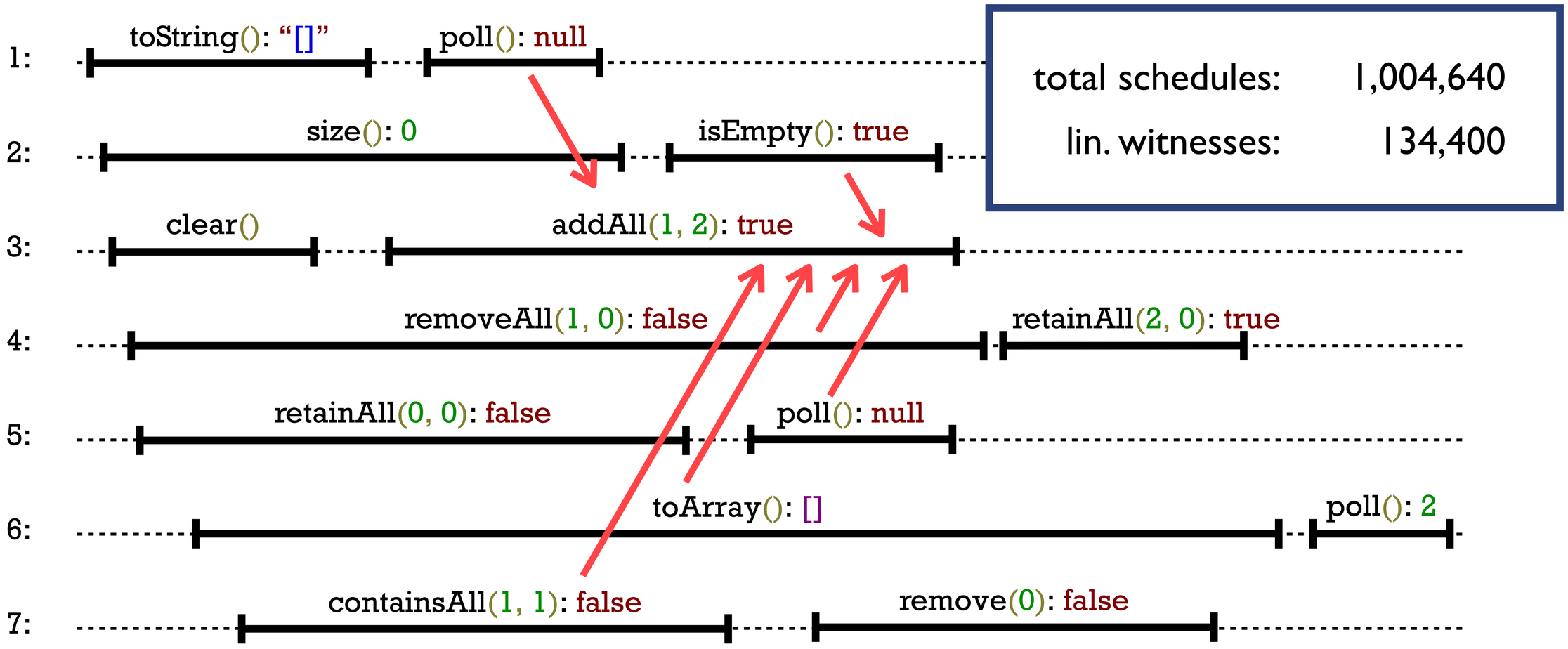
d-Linearizable history:

There exist operations $\mathbf{op_0, \ldots, op_{d-1}}$ such that every schedule that strongly hits $\mathbf{(op_0, \ldots, op_{d-1})}$ is a witness to linearizability

Linearizability depth of a history:

Smallest $\mathbf{d \geq 1}$ such that the history is **d**-linearizable

History from the example has linearizability depth 1



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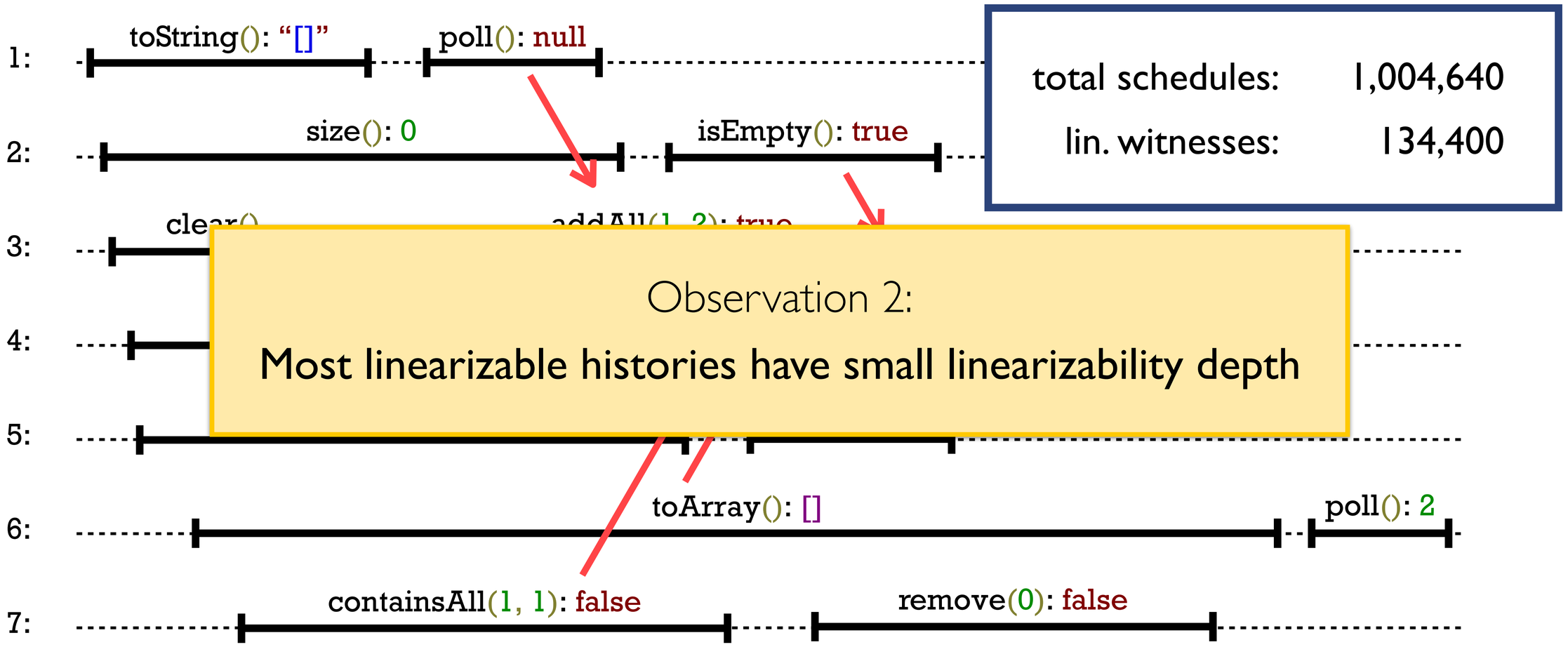


Checking d-linearizability

Recall: A history is **d**-linearizable if there exist operations $\mathbf{op_0}, \ldots, \mathbf{op_{d-1}}$ such that every schedule that strongly hits $\mathbf{(op_0}, \ldots, \mathbf{op_{d-1})}$ is a witness to linearizability

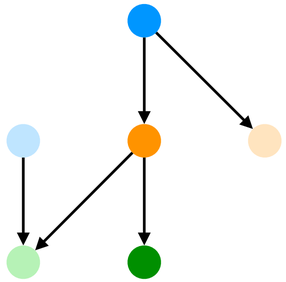
Strong d-hitting family: A set of schedules $\mathcal{F}$ is a **strong d-hitting family** if it strongly hits every **d**-tuple of operations

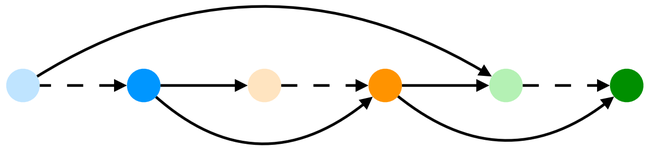
Conclusion: To check **d**-linearizability, it suffices to explore schedules from a strong **d**-hitting family

Size of strong hitting families [OOPSLA '18]

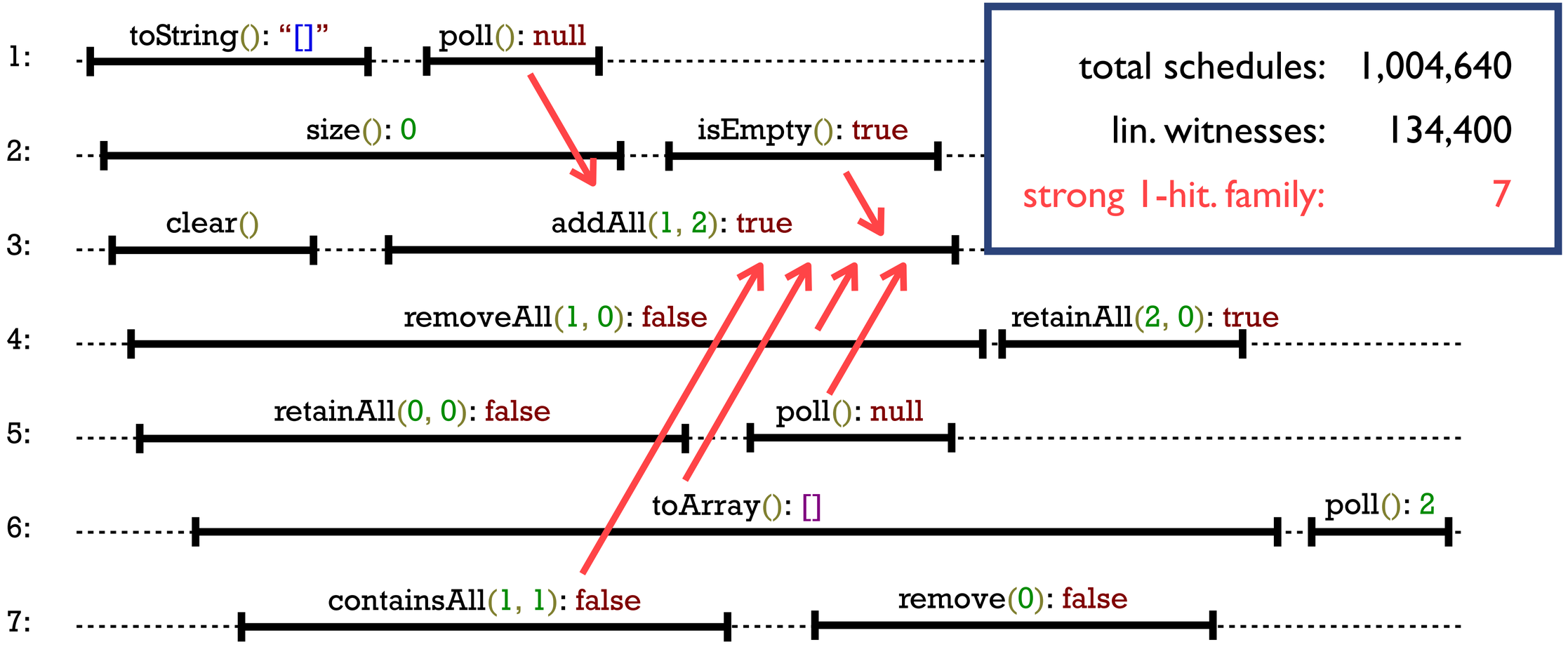
Given a history of **n** operations on **k** threads, and **d ≥ 1**:

- There is a strong **d**-hitting family of size **$O(n^d)$**
- There is a strong **d**-hitting family of size **$O(kn^{d-1})$**:
  A single schedule can strongly hit all operations in a thread





For the history from the example, 1-linearizability is shown by exploring 7 schedules!



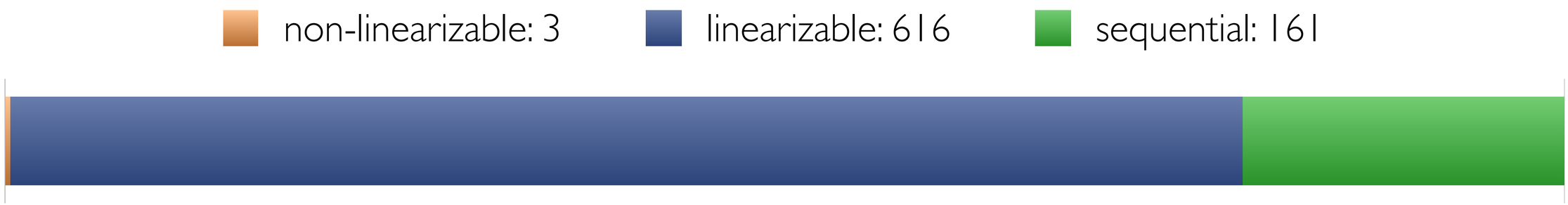
Experiments

Observation 1: Most execution histories are linearizable

Observation 2: Most linearizable histories have small linearizability depth

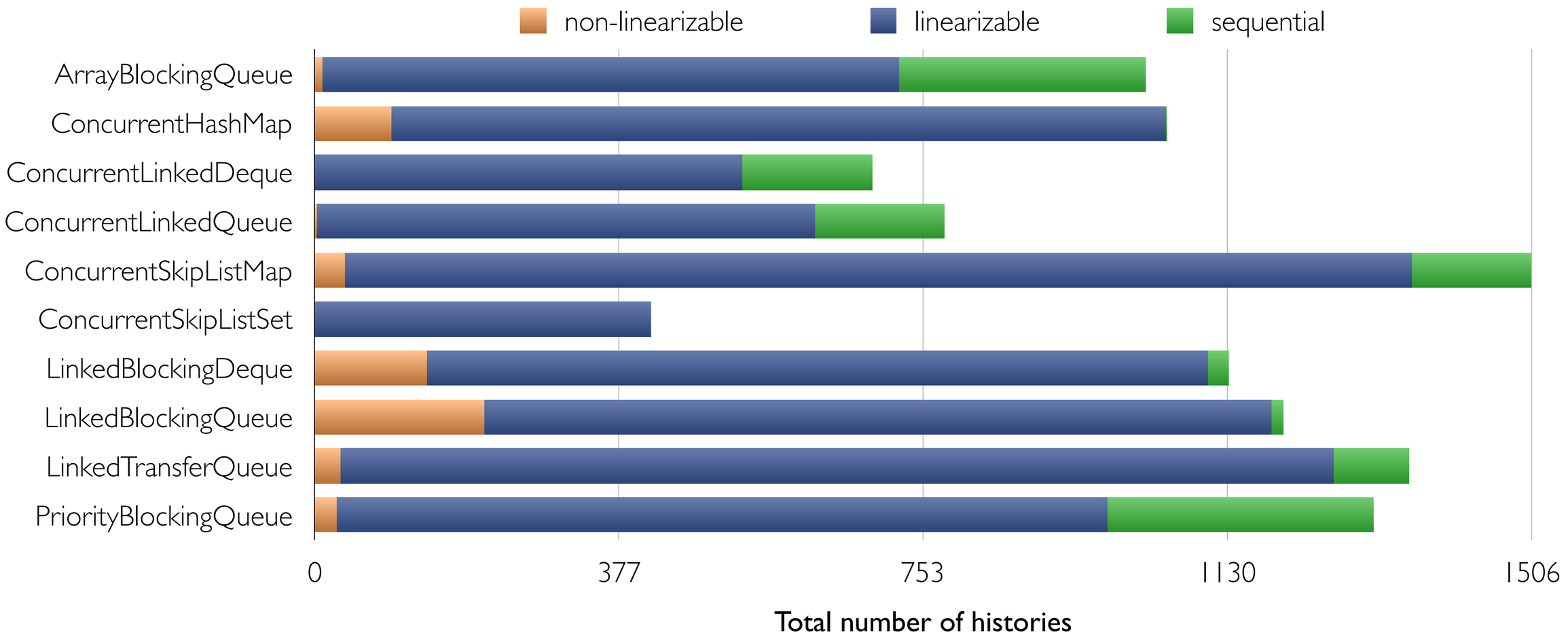
Goal:
Validate the observations for the histories generated by Violat on **java.util.concurrent**

Breakdown of histories generated by Violat for ConcurrentLinkedQueue

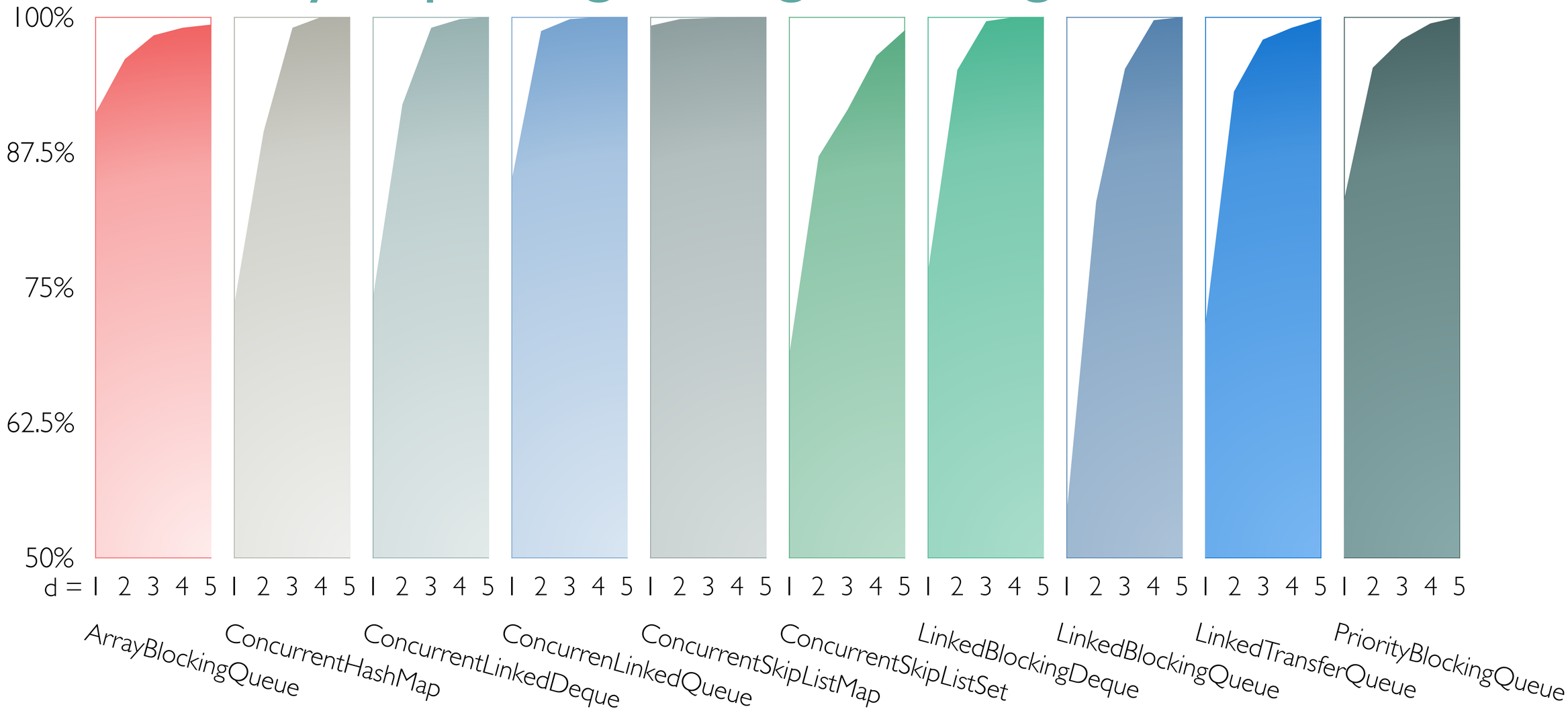


Total: 780 histories

Breakdown of histories generated by Violat



Linearizable histories whose linearizability is shown by exploring strong d-hitting families



Conclusion

Contributions

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Future work

- Optimized linearizability checker for **Jepsen**; experiments on distributed systems
- Weaker correctness conditions like causal consistency